Name: KEY

Section 2.3

1. Divide using long division

a) $\left(6x^3-16x^2+17x-6\right)\div\left(3x-2\right)$

$$= 2x^2-4x+3$$

$$\begin{array}{r}
2x^2-4x+3 \\
3x-2\,\overline{)\,6x^3-16x^2+17x-6} \\
-(6x^3-4x^2) \\
\hline
-12x^2+17x \\
-(-12x^2+8x) \\
\hline
9x-6 \\
9x-6 \\
\hline
0
\end{array}$$

b) $\left(x^3-2x^2+3x-9\right)\div\left(x^2+1\right)$

$$= x-2+\frac{2x-7}{x^2+1}$$

$$\begin{array}{r}
x-2 \\
x^2+1\,\overline{)\,x^3-2x^2+3x-9} \\
-(x^3+x) \\
\hline
-2x^2+2x-9 \\
-(-2x^2-2) \\
\hline
2x-7
\end{array}$$

2. Using synthetic division, dived by the expression $(x-k)$. Then express the function in the form $f(x)=(x-k)q(x)+r$

a) $f(x)=x^3-5x^2-11x+8;\quad k=-2$

$$\begin{array}{r|rrrr}
-2 & 1 & -5 & -11 & 8 \\
 & & -2 & 14 & -6 \\
\hline
 & 1 & -7 & 3 & 2
\end{array}$$

$$x^2-7x+3+\frac{2}{x+2}$$

b) $f(x) = 10x^3 - 22x^2 - 3x + 4;\quad k = \frac{1}{5}$

$$\begin{array}{r|rrrr}
\frac{1}{5} & 10 & -22 & -3 & 4 \\
 & & 2 & -4 & -7/5 \\
\hline
 & 10 & -20 & -7 & 13/5
\end{array}$$

$$\frac{f(x)}{x-\frac{1}{5}} = 10x^2-20x-7+\frac{13/5}{x-1/5}$$

Use synthetic division to find each function value. To check, verify your answer by substitution.

3. $g(x)=x^6-4x^4+3x^2+2$

a. $g(2)$

2	1	0	-4	0	3	0	2
		2	4	0	0	6	12
	1	2	0	0	3	6	14

$g(2)=14$

b. $g(-4)$

-4	1	0	-4	0	3	0	2
		-4	16	-48	192	-780	3120
	1	-4	12	-48	195	-780	3122

$g(-4)=3122$

4. $g(x) = 3x^3 + 5x^2 - 10x + 1$

a. $g(3)$

3	3	5	-10	1
		9	42	96
	3	14	32	97

$g(3)=97$

b. $g(-2)$

-2	3	5	-10	1
		-6	2	16
	3	-1	-8	17

$g(-2)=17$

5. You are given x as a zero of the polynomial function. Use synthetic division to verify, then use the result to factor the polynomial completely. List all real zeros of the function.

$f(x)=x^3-28x-48=0;\ x=-4$

-4	1	0	-28	-48
		-4	16	48
	1	-4	-12	0

$f(x)=(x+4)(x^2-4x-12)=(x+4)(x-6)(x+2)$

zeros: $-4, -2, 6$

6. Use the *zero* feature of your calculator to approximate all the zeros of the function to three decimal places. Determine one of the exact zeros, use synthetic division to verify your result, and then completely fact the polynomial.

$g(x) = x^3 - 4x^2 - 2x + 8$

Zeros: $-1.414, 1.414, 4$

$$\begin{array}{r|rrrr} 4 & 1 & -4 & -2 & 8 \\ & & 4 & 0 & -8 \\ \hline & 1 & 0 & -2 & 0 \end{array}$$

$g(x) = (x-4)(x^2-2) = (x-4)(x+\sqrt{2})(x-\sqrt{2})$

Zeros: $4, \pm\sqrt{2}$

Section 2.4

7. Find numbers a and b such that the equation is true.

a) $(a - 1) + (b + 3)i = 5 + 8i$

$a - 1 = 5$ $b + 3 = 8$

$a = 6$ $b = 5$

b) $(a + 6) + 2bi = 6 - 5i$

$a + 6 = 6$ $2b = -5$

$a = 0$ $b = -5/2$

Simplify the following expressions completely. Express your answers as a complex number

8. $\left(-2+\sqrt{-8}\right)+\left(5-\sqrt{-50}\right)$

$-2+i\sqrt{8}+5-i\sqrt{50}$

$-2+2i\sqrt{2}+5-5i\sqrt{2}$

$3-3i\sqrt{2}$

9. $\left(4+7i\right)^2$

$16+28i+28i+49i^2$

$16+56i-49$

$-33+56i$

10. $(8+\sqrt{18})-(4+i\sqrt{12})$

$8+3\sqrt{2}-4-2i\sqrt{3}$

$4+3\sqrt{2}-2i\sqrt{3}$

11. $(9-2i)^2+(6+4i)$

$81-18i-18i+4i^2+6+4i$

$87-32i-4$

$83-32i$

12. $\left(8-3i\right)\left(5+2i\right)$

$40+16i-15i-6i^2$

$46+i$

13. Write the quotient in standard form.

a) $\frac{6-i}{3+2i}\cdot\frac{3-2i}{3-2i}=\frac{18-12i-3i+2i^2}{9-4i^2}$

$=\frac{16-15i}{13}$

b) $\frac{(8+16i)}{0+2i}\cdot\frac{0-2i}{0-2i}=\frac{-16i-32i^2}{-4i^2}$

$=\frac{32-16i}{4}=8-4i$

14. Use the quadratic formula to solve the quadratic equation.

a) $9x^2-6x+37=0$

$=\frac{-(-6)\pm\sqrt{(-6)^2-4(9)(37)}}{2(9)}$

$=\frac{6\pm\sqrt{-1296}}{18}=\frac{6\pm 36i}{18}$

$=\frac{1\pm 6i}{3}$

b) $4x^2+16x+17=0$

$=\frac{-16\pm\sqrt{16^2-4(4)(17)}}{2(4)}$

$=\frac{-16\pm\sqrt{-16}}{8}=\frac{-16\pm 4i}{8}$

$=-2\pm\frac{i}{2}$

Section 2.5

Use the Rational Zero Theorem to give a listing of all *possible* rational zeros. Then, graph the function on your calculator to find the *actual* zeros.

15. $f(x) = 4x^5 - 8x^4 - 5x^3 + 10x^2 + x - 2$

$p: \pm(1, 2)$
$q: \pm(1, 2, 4)$

Possible rational zeros: $\frac{\pm(1,2)}{\pm(1,2,4)} = \pm(1, 2, \frac{1}{2}, \frac{1}{4})$

Actual zeros: $-1, -\frac{1}{2}, \frac{1}{2}, 1, 2$

16. You are given x as a zero of the polynomial equation. Verify using synthetic division, then use the result to factory the polynomial completely. List all real zeros of the function.

$f(x) = 2x^3 + x^2 - 5x + 2 = 0;\ x = -2$

$$\begin{array}{r|rrrr} -2 & 2 & 1 & -5 & 2 \\ & & -4 & 6 & -2 \\ \hline & 2 & -3 & 1 & 0 \end{array}$$

$f(x) = (x+2)(2x^2 - 3x + 1)$
$= (x+2)(2x-1)(x-1)$

Zeros: $x = -2, +\frac{1}{2}, 1$

For questions 17-22, a) List the *possible* rational zeros of $f(x)$, b) Use your calculator to find an exact zero, and c) use synthetic division to determine all real zeros of $f(x)$.

17. $f(x) = x^3 - 9x^2 + 27x - 27$

possible: $\frac{\pm(1, 3, 9, 27)}{\pm 1}$

$$\begin{array}{r|rrrr} 3 & 1 & -9 & 27 & -27 \\ & & 3 & -18 & 27 \\ \hline & 1 & -6 & 9 & 0 \end{array}$$

$f(x) = (x-3)(x^2 - 6x + 9)$
$= (x-3)(x-3)(x-3)$

Zeros: $x = 3$

18. $f(x) = x^3 - 9x^2 + 20x - 12$

possibles: $\pm(1, 2, 3, 4, 6, 12)$

$$\begin{array}{r|rrrr} 2 & 1 & -9 & 20 & -12 \\ & & 2 & -14 & 12 \\ \hline & 1 & -7 & 6 & 0 \end{array}$$

$f(x) = (x-2)(x^2-7x+6)$

$= (x-2)(x-6)(x-1)$

zeros: $x = 1, 2, 6$

19. $f(x) = x^4 - 13x^2 - 12x$

$f(x) = x(x^3 - 13x - 12)$

possibles: $\pm(1, 2, 3, 4, 6, 12)$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -13 & -12 \\ & & -1 & 1 & 12 \\ \hline & 1 & -1 & -12 & 0 \end{array}$$

$f(x) = x(x+1)(x^2 - x - 12)$

$= x(x+1)(x-4)(x+3)$

zeros: $x = 0, -1, 4, -3$

20. $f(x) = -3x^3 + 20x^2 - 36x + 16$

possibles: $\dfrac{\pm(1, 2, 4, 8, 16)}{\pm(1, 3)}$

$$\begin{array}{r|rrrr} 2 & -3 & 20 & -36 & 16 \\ & & -6 & 28 & -16 \\ \hline & -3 & 14 & -8 & 0 \end{array}$$

$f(x) = (x-2)(-3x^2 + 14x - 8)$

$= -(x-2)(3x^2 - 14x + 8)$

$= -(x-2)(3x-2)(x-4)$

zeros: $x = 2, \frac{2}{3}, 4$

21. $f(x) = 4x^4 - 17x^2 + 4$

possibles: $\frac{\pm(1,2,4)}{\pm(1,2,4)} = \pm(1, 2, 4, \frac{1}{2}, \frac{1}{4})$

$$\begin{array}{r|rrrrr} 2 & 4 & 0 & -17 & 0 & 4 \\ & & 8 & 16 & -2 & -4 \\ \hline & 4 & 8 & -1 & -2 & 0 \end{array}$$

$(x-2)$

$$\begin{array}{r|rrrr} \frac{1}{2} & 4 & 8 & -1 & -2 \\ & & 2 & 5 & 2 \\ \hline & 4 & 10 & +4 & 0 \end{array}$$

$(x-2)(x-\frac{1}{2})(4x^2+10x+4)$

$(x-2)(x-\frac{1}{2})(2)(2x^2+5x+2)$

$\cancel{2}(x-2)(x-\frac{1}{2})(2x+1)(x+2)$

zeros: $x = 2, \frac{1}{2}, -\frac{1}{2}, -2$

22. $f(x) = x^4 - 7x^2 + 12$

Possibles: $\pm(1, 2, 3, 4, 6, 12)$

$$\begin{array}{r|rrrrr} -2 & 1 & 0 & -7 & 0 & 12 \\ & & -2 & 4 & 6 & -12 \\ \hline & 1 & -2 & -3 & 6 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & -3 & 6 \\ & & 2 & 0 & -6 \\ \hline & 1 & 0 & -3 & 0 \end{array}$$

$(x+2)(x-2)(x^2-3) = f(x)$

zeros: $x = 2, -2, \pm\sqrt{3}$

Application Problems!

23. A box is to be mailed. The volume in cubic inches of the box can be expressed as the product of its three dimensions: $V(x) = x^3 - 16x^2 + 79x - 120$. The length is $(x - 8)$. Find linear expressions for the other dimensions. Assume that the width is greater than the height.

$$\begin{array}{r|rrrr} 8 & 1 & -16 & 79 & -120 \\ & & 8 & -64 & 120 \\ \hline & 1 & -8 & 15 & 0 \end{array}$$

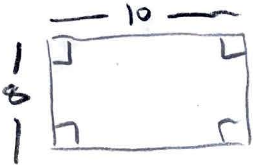
$(x-8)(x^2-8+15)$

$(x-8)(x-3)(x-5)$

l · w · h

length = $x-8$
width = $x-3$
height = $x-5$

24. An open box is made from an 8-by-10 inch rectangular piece of cardboard by cutting squares from each corner and folding up the sides. If x represents the side lengths of the squares, write a function giving the volume $V(x)$ of the box in terms of x.



$V(x) = x(10-2x)(8-2x)$

25. A polynomial function, $f(x) = x^4 - 5x^3 - 28x^2 + 188x - 240$, is used to model a new roller coaster section. The loading zone will be placed at one of the zeros. The function has a zero at 5. What are the possible locations for the loading zone? What methods can you use to solve this problem? Solve it!!

5	1	-5	-28	188	-240
		5	0	-140	240
	1	0	-28	48	0

4	1	0	-28	48
		4	16	-48
	1	4	-12	0

$(x-5)(x-4)(x^2+4x-12)$

$(x-5)(x-4)(x+6)(x-2) = f(x)$

loading zone @ 5, 4, -6, 2